

A dynamic Stackelberg game for green supply chain management

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Abstract

In this paper, we establish a dynamic game to allocate CSR (Corporate Social Responsibility) to the members of a supply chain. We propose a model of a three-tier supply chain in a decentralized state which includes a supplier, a manufacturer and a retailer. For analyzing supply chain performance in decentralized state and the relationships between the members of the supply chain, we use a Stackelberg game and consider in this paper a hierarchical equilibrium solution for a two-level game. In particular, we formulate a model that crosses through multi-periods with the help of a dynamic discrete Stackelberg game. We obtain an equilibrium point at which both the profits of members and the level of CSR taken up by supply chains is maximized.

Keywords: Dynamic Game; Supply Chain; CSR; Stackelberg Game.

1 Introduction

In recent years, companies and firms have been showing an ongoing interest in favor of CSR. This is mainly because of increasing consumer awareness of several CSR issues, e.g. the environment, human rights and safety. In addition, the firms are also forced to accept CSR due to government policies and regulations. Recently CSR has gained recognition and importance as field of research

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field [8, 15]. However, the research field still lacks a consistent definition of CSR and this has been the center of discussion since several decades. Dahlsrud [9] presented an overview of different definitions of CSR and summarized the number of dimensions included in each definition. There is a positive correlation between CSR and profit [19, 20]. Moreover, CSR is an effective tool for supply chain management, for coordination, purchasing, manufacturing, distribution, and marketing functions [14]. According to previous studies, the long term investment on CSR is beneficial for a supply chain. Furthermore, a sustainable supply chain requires consideration of the social aspects of the business [23]. Carter et al. [6] established an effective approach and demonstrated that environmental purchasing is significantly related to both net income and cost of goods sold. Carter et al. [7] also pointed towards the importance of CSR in the supply chain, in particular the role played by the purchasing managers in socially responsible activities and the effect of these activities on the supply chain. Sethi [21] introduced a taxonomy in which a firm's social activities include social obligations as well as more voluntary social responsibility. And, Carroll [4, 5] developed a framework for CSR that consists of economic, legal, and ethical responsibilities.

The members of a supply chain take their decisions based on maximizing their individual net benefits. In addition, when they need to accept CSR; this situation leads to an equilibrium status. Game theory is one of the most effective tools to deal with this kind of management problems.

A growing number of research papers use game theoretical applications in supply chain management. Cachon et al. [3] discuss Nash equilibrium in noncooperative cases in a supply chain with one supplier and multiple retailers. Hennessey et al. [13] presented a paper to evaluate the efficiency of different types of contracts between the industrial partners of a supply chain. They applied game theory methods for decisional purposes. Tian et al. [24] presented a system dynamics model based on evolutionary game theory for green supply chain management.

In this paper, we consider a discrete time version of the dynamic differential game. The optimal control theory is the standard tool for analyzing the differential game theory [16]. There are two different types of information structures in a differential game, open-loop and feedback information structures. In an open-loop strategy, the players choose their decisions at time t , with information of the state at time zero. In contrast, in a feedback information structure, the players use their knowledge of the current state at time t in order to formulate their decisions at time t [11]. We formulate a model and study the behavior for decentralized supply chain networks under CSR conditions with one leader and two followers. The Stackelberg game model is recommended and applied here to find an equilibrium point at which the profit of the members of the supply chain is maximized and the level of CSR is adopted in the supply chain. We develop an open-loop Stackelberg game by selecting the supplier as the leader and both the manufacturer and the retailer as the followers. Using this approach, the supplier as a leader, can know the optimal reaction of his followers, and utilizes such processes to maximize his own profit. The manufacturer and the retailer as followers, try to maximize their profits by considering all the conditions. Our model has two levels, at the first level the manufacturer is the leader and the retailer is the follower and we find the equilibrium point. At the second level, we consider the supplier as the leader and the manufacturer as the follower. In fact, we substitute the response functions of the follower into the objective function of the leader and we find the final equilibrium point. We propose a Hamiltonian matrix to solve the optimal control problem to obtain the equilibrium in this game. The paper is organized as follows: Section 2 is devoted to the problem description and assumptions. Objective functions, constraints and solution of the game are illustrated in Section 3. A numerical example is shown in Section 4 and we close with a conclusion in Section 5.

2 Problem Description and Assumptions

We consider a three stage Stackelberg differential game which has three players playing the game over a fixed finite horizon model. This model is a three-tier, decentralized vertical control supply chain network (Figure 1). All retailers and suppliers at the same level make the same decision. Therefore, consequently the model has only one supplier, one manufacture and one retailer. The simplified model is shown in Figure 2.

The dynamic game goes through multi-periods as a repeated game with complete information. This model has a state variable and control variables like any dynamic game. We define the state variable as the level of social responsibility taken up by companies, and the control variables are the capital amounts invested while fulfilling the social responsibility. Specifically, all of the social responsibilities taken up by the firm j at period t can be expressed as the investment I_t^j . We suppose that x_t evolves according to the following rule: $x_{t+1} = f(I_t, x_t)$.

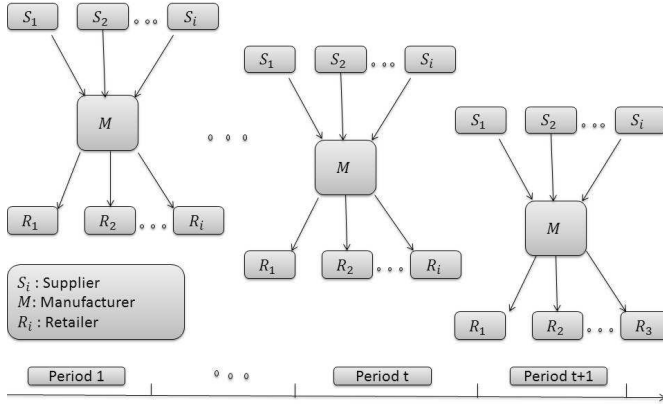


Figure 1: Three-tier supply chain network

More specifically we have the following assumptions:

The function $B_t(x_t) = \delta x_t$ represents social benefit which is proportional to social responsibility taken up by the supply chain system [1].

The function $T_t = \tau I_t [1 + \theta(I_t)]$ measures the value of the tax return to the members of the supply chain [10]. Both τ and θ are tax return policy parameters. Specifically, τ is the rate of individual post tax return on investment (ROI), and θ is the rate of supply chain's post tax return on investment (ROI).

The market inverse demand is $P^M(q_t) = a - bq_t$ [17].

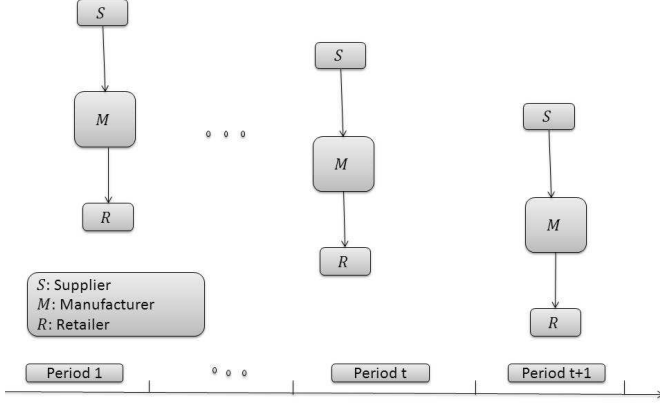


Figure 2: Simplified model of three-tier supply chain network

The accumulation of the level of social responsibility taken up by the firms is given by $x_{t+1} = \alpha x_t + \beta_1 I_t^S + \beta_2 I_t^M + \beta_3 I_t^R$.

Here, β_1 is the rate of converting the supplier's capital investment in CSR to the amount of CSR taken up by the supply chain; β_2 is the rate of converting the manufacturer's capital investment in CSR to the amount of CSR taken up by the supply chain and β_3 is the rate of converting the retailer's capital investment in CSR to the amount of CSR taken up by the supply chain as well [22].

2.1 The General Framework

He et al. [12] illustrate an open-loop Stackelberg differential game model over a fixed finite horizon time as detailed in the following:

The follower's optimal control problem is:

$$Max_{r(\cdot)} \left\{ J_R(X_0, r(\cdot); w(\cdot)) = \int_0^t e^{-\rho t} \pi_R(X(t), w(t), r(t)) dt \right\}, \quad (2.1)$$

subject to the state equation

$$\begin{aligned} \dot{X}(t) &= F(X(t), w(t), r(t)), \\ X(0) &= X_0. \end{aligned} \quad (2.2)$$

where the function F represents the rate of sales, ρ is the followers's discount rate, and X_0 , is the initial condition. The follower's Hamiltonian is

$$H_R(X, r, \lambda_R, w) = \pi_R(X, w, r) + \lambda_R F(X, w, r), \quad (2.3)$$

where λ_R is the vector of the shadow prices associated with the state variable X ; and it satisfies the adjoint equation

$$\dot{\lambda}_R = \rho \lambda_R - \frac{\partial H_R(X, r, \lambda_R, w)}{\partial X}, \quad \lambda_R(T) = 0. \quad (2.4)$$

The necessary optimality condition for the follower's problem satisfies

$$\frac{\partial H_R}{\partial r} = 0 \implies \frac{\partial \pi_R(X, w, r)}{\partial r} + \lambda_R \frac{\partial F(X, w, r)}{\partial r} = 0. \quad (2.5)$$

We derive the follower's best response $r^*(X, w, \lambda_R)$.

The leader's problem is

$$\begin{aligned} Max_{w(\cdot)} & \left\{ J_M(X_0, w(\cdot)) = \int_0^t e^{-\mu t} \pi_M(x, w, r(x, w, \lambda_R)) dt \right\}, \\ \dot{X} &= F(X, w, r(x, w, \lambda_R)), \\ X_0(0) &= X_0, \\ \dot{\lambda}_R &= \rho \lambda_R - \frac{\partial H_R(x, r(x, w, \lambda_R), \lambda_R, w)}{\partial x}, \quad \lambda_R(T) = 0, \end{aligned} \quad (2.6)$$

where μ is the leader's discount rate and the above differential equations are obtained by substituting the follower's best response $r^*(X, w, \lambda_R)$ into the state equation and the adjoint equation of the follower, respectively. We formulate the leader's Hamiltonian as follows

$$H_M = \pi_M(x, \lambda_R, w, r(X, w, \lambda_R), \lambda_M, \varphi) + \lambda F(X, w, r(X, w, \lambda_R)) - \mu \frac{\partial H_R(X, r(X, w, \lambda_R), \lambda_R, w)}{\partial X}, \quad (2.7)$$

where λ_M and μ are the shadow associated with X and λ_R , respectively, and they satisfy the adjoint equations

$$\begin{aligned} \dot{\lambda}_M &= \mu \lambda_M - \frac{\partial H_M(X, \lambda_R, w, r(X, w, \lambda_R), \lambda_M, w)}{\partial X} \\ &= \mu \lambda_M - \frac{\partial \pi_M(X, w, r(X, w, \lambda_R))}{\partial X} \\ &\quad - \lambda_M \frac{\partial F(X, w, r(X, w, \lambda_R))}{\partial X} - \mu \frac{\partial^2 H_R(x, r(X, w, \lambda_R), \lambda_R, w)}{\partial X^2}, \\ \dot{\varphi} &= \mu \varphi - \frac{\partial H_M(X, \lambda_R, w, r(X, w, \lambda_R), \lambda_M, \varphi)}{\partial \lambda_R} \\ &= \mu \varphi - \lambda_M \frac{\partial F(x, w, r(X, w, \lambda_R))}{\partial \lambda_R} - \mu \frac{\partial^2 H_R(X, r(X, w, \lambda_R), \lambda_R, w)}{\partial \lambda \partial X}, \end{aligned} \quad (2.8)$$

where $\lambda_M(T) = 0$ and $\varphi(0) = 0$ are the boundary conditions.

We apply the algorithm of the above general model as part of our model.

2.2 Notations and Definitions

To facilitate the model, certain parameters and decision variables are used.

Table 1 shows notations and definitions that we use in our model.

Variables	
t	Period t
T	Planning horizon
q_t	Demand quantity at period t
a	Market potential
b	Price sensitivity
x_t	State variable, degree of taking SR
H^S	Hamiltonian function of the supplier
H^M	Hamiltonian function of the manufacturer
H^R	Hamiltonian function of the retailer
J_t^S	Objective function of the supplier
J_t^M	Objective function of the manufacturer
J_t^R	Objective function of the retailer
$B^M(x_t)$	Social benefit of the supplier
$B^S(x_t)$	Social benefit of the manufacturer
$B^R(x_t)$	Social benefit of the retailer
$T^S(x_t)$	Tax return of the supplier
$T^M(x_t)$	Tax return of the manufacturer
$T^R(x_t)$	Tax return of the retailer
I_t^M	The amount of investment done by the manufacturer
I_t^S	The amount of investment done by the supplier
I_t^R	The amount of investment done by the retailer
d	The percentage of investment of the supplier payoff
\hat{d}	The percentage of investment of the manufacturer payoff
w	Price of the supplier's raw material
z	Price of sold product by the retailer
δ	Parameter of the supplier's social benefit
$\hat{\delta}$	Parameter of the manufacturer's social benefit
$\hat{\hat{\delta}}$	Parameter of the retailer's social benefit
λ	Quantity discount parameter of the price of raw material
α	Deteriorating rate of the level of current social responsibility
τ	The rate of individual post tax return on investment (ROI)
θ	The rate of supply chain's post tax return on investment (ROI)
β_1	The rate of converting the supplier's capital investment in CSR to the amount of CSR taken up by the supply chain
β_2	The rate of converting the manufacturer's capital investment in CSR to the amount of CSR taken up by the supply chain
β_3	The rate of converting the retailer's capital investment in CSR to the amount of CSR taken up by the supply chain

Table 1: Notations and Definitions.

3 Objective Functions and Constraints

The objective functions are made to depend on the control vectors and the static variable. The members of the supply chain attempt to optimize their net profits, which includes minimizing the cost of raw materials and investment in social responsibility, and maximizing sale revenues and

benefits from taking social responsibility as well as tax returns. Thus, the objective function of the supplier is

$$\begin{aligned} J^S &= \sum_{t=1}^T P_t^S q_t - c q_t + B_t^S(x_t) + T_t^S(I_t^S, I_t) - I_t^S + d I_t^M \\ &= \sum_{t=1}^T w q_t - c q_t + \delta x_t^2 + \tau I_t^S [1 + \theta(I_t^S + I_t^M + I_t^R)] - I_t^S + d I_t^M, \end{aligned}$$

where, P_t^S is the price of the supplier's raw material. $P_t^S = w$. $B_t^S(x_t)$ is the social benefit of the supplier, δ is the parameter of the supplier's social benefit and $T_t^S(I_t^S, I_t)$ is the tax return of the supplier. Similarly, the objective function of the manufacturer is

$$\begin{aligned} J^M &= \sum_{t=1}^T P_t^M(q_t) q_t - P_t^S q_t + B_t^M(x_t) + T_t^M(I_t^M, I_t) - I_t^M + \widehat{d} I_t^R \\ &= \sum_{t=1}^T (a - b q_t) q_t - w q_t + \widehat{\delta} x_t^2 + \tau I_t^M (1 + \theta(I_t^S + I_t^M + I_t^R)) - I_t^M + \widehat{d} I_t^R, \end{aligned}$$

where $P_t^M(q_t)$ is the retail price of the product of the manufacturer. $B_t^M(x_t)$ is the social benefit of the manufacturer, $\widehat{\delta}$ is the parameter of the manufacturer's social benefit and $T_t^M(I_t^M, I_t)$ is the tax return of the manufacturer.

The objective function of the retailer is

$$\begin{aligned} J^R &= \sum_{t=1}^T P_t^R q_t - P_t^M(q_t) q_t + B_t^R(x_t) + T_t^R(I_t^R, I_t) - I_t^R \\ &= \sum_{t=1}^T z q_t - (a - b q_t) q_t + \widehat{\widehat{\delta}} x_t^2 + \tau I_t^R (1 + \theta(I_t^S + I_t^M + I_t^R)) - I_t^R, \end{aligned}$$

where P_t^R is the price at which the retailer sells the product to the consumer. $P_t^R = Z$. $B_t^R(x_t)$ is the social benefit of the retailer, $\widehat{\widehat{\delta}}$ is the parameter of the retailer's social benefit and $T_t^R(I_t^R, I_t)$ is the tax return of the retailer.

3.1 Mathematical Model: Level One

At this level, we establish a Stackelberg game between the manufacturer as the leader and the retailer as the follower. To calculate the equilibrium at this level, first we calculate the best reaction function of the retailer, then we determine the manufacturer's optimal decisions based on the retailer's best reactions.

Since our dynamic differential game is an optimal control problem, we can apply the Hamiltonian function to find the equilibrium of the game [21].

Suppose the time interval is $[1, T]$. For any fixed I_t^S and I_t^M the retailer solves

$$\arg \max_{I_t^R} \sum_{t=1}^T P_t^R q_t - P_t^M(q_t) q_t + B_t^R(x_t) + T_t^R(I_t^R, I_t) - I_t^R,$$

subject to $x_{t+1} = \alpha x_t + \beta_1 I_t^S + \beta_2 I_t^M + \beta_3 I_t^R$.

We define the retailer's Hamiltonian for fixed I_t^S and I_t^M as

$$H_t^R = J_t^R + P_{t+1}^R(x_{t+1}).$$

By using the conditions for a maximization of this Hamiltonian, we compute:

$$I_t^R = \frac{1 - P_{t+1}^R \beta_3 - \tau \theta (I_t^M + I_t^S) - \tau}{2\theta \tau}. \quad (3.1)$$

The equation of I_t^R which depends on I_t^S and I_t^M , says that for any given strategy of I_t^S and I_t^M , there is a unique optimal response I_t^R .

$$x_{t+1} = \frac{\partial H_t^R}{\partial P_{t+1}^R} = \alpha x_t + \beta_1 I_t^S + \beta_2 I_t^M + \beta_3 I_t^R, \quad (3.2)$$

and by substituting (3.1) in (3.2), we obtain

$$x_{t+1} = (\beta_1 - \beta_3/2)I_t^S + (\beta_2 - \beta_3/2)I_t^M + \alpha x_t + \beta_3 \frac{1 - P_{t+1}^R \beta_3 - \tau}{2\tau \theta}. \quad (3.3)$$

We also have

$$P_t^R = \frac{\partial H_t^R}{\partial x_t} = 2\hat{\delta}x_t + \alpha P_{t+1}^R. \quad (3.4)$$

The above sets of equations define the reaction function of the retailer.

For any fixed I_t^S the manufacturer solves

$$\arg \max_{I_t^M} \sum_{t=1}^T P_t^M(q_t)q_t - P_t^S q_t + B^M(x_t) + T^M(I_t^M, I_t) - I_t^M + \hat{d}I_t^R,$$

subject to $x_{t+1} = \alpha x_t + \beta_1 I_t^S + \beta_2 I_t^M + \beta_3 I_t^R$.

Now, we substitute the value of I_t^R in (3.1) into J_t^M , and we obtain

$$J_t^M = (a - bq_t)q_t - wq_t + \hat{\delta}x_t^2 + \frac{\tau\theta - \hat{d}}{2}I_t^S I_t^M + \frac{\tau - 1 - \beta_3 P_{t+1}^R - \hat{d}}{2}I_t^M - \frac{\hat{d}}{2\theta}. \quad (3.5)$$

The Hamiltonian function of the manufacturer for fixed I_t^S is

$$H_t^M = J_t^M + P_{t+1}^M(x_{t+1}) + u_t(P_t^R), \quad (3.6)$$

consequently, we can obtain the unique optimal response of the follower from the equations as follows.

$$\frac{\partial H_t^M}{\partial I_t^M} = \tau (1 + \theta ((1 - \tau\theta)I_t^S + I_t^M)) - 1 + \frac{1 - \beta_3 P_{t+1}^R - \tau - \tau \hat{d}}{2} + (\beta_2 - \beta_3/2) P_{t+1}^M,$$

and we get

$$I_t^M = \frac{-\tau\theta I_t^S + \beta_3 P_{t+1}^R + (1 + \hat{d} - \tau)}{2\tau\theta} - \frac{(\beta_2 - \beta_3/2) P_{t+1}^M}{\tau\theta}. \quad (3.7)$$

Other constraints are

$$P_t^M = \frac{\partial H_t^M}{\partial x_t} = 2\hat{\delta}x_t + \alpha P_{t+1}^M + 2\hat{\delta}u_t, \quad (3.8)$$

$$u_{t+1} = \frac{\partial H_t^M}{\partial P_{t+1}^R} = -\beta_3/2 I_t^M - \frac{\hat{d}\beta_3}{2\tau\theta} - \frac{P_{t+1}^M \beta_3^2}{2\tau\theta} + \alpha u_t. \quad (3.9)$$

The equation of I_t^M depends on I_t^S , and we obtain the final equilibrium in the next section, at level two.

3.2 Mathematical Model: Level Two

At the previous level, the manufacturer's optimal function was calculated by using a reaction function of the retailer. At this level, the reaction functions of two followers (retailer and manufacturer) are inserted into the objective function of the leader (supplier), and we can find the final equilibrium point.

The problem facing the supplier is simply given by

$$\arg \max_{I_t^S} \sum_{t=1}^T P_t^S q_t - c q_t + B_t^S(x_t) + T_t^S(I_t^S, I_t) - I_t^S + d I_t^M,$$

subject to $x_{t+1} = \alpha x_t + \beta_1 I_t^S + \beta_2 I_t^M + \beta_3 I_t^R$.

The Hamiltonian function of the supplier is defined by

$$H_t^S = J_t^S + P_{t+1}^S(x_{t+1}) + \mu_t(P_t^M) + u_t(p_t^R). \quad (3.10)$$

Substitute (3.1) and (3.7) into (3.10), we get the value of I_t^S , x_{t+1} and μ_{t+1}

$$\frac{\partial H_t^S}{\partial I_t^S} = 0, \quad (3.11)$$

therefore

$$I_t^S = \frac{(\beta_3/2 + \beta_2 - 2\beta_1)}{\tau\theta} P_{t+1}^S + \frac{(\beta_2 - \beta_3/2)}{\tau\theta} P_{t+1}^M + \frac{\beta_3}{2\tau\theta} P_{t+1}^R + \frac{3 - 3\tau - \widehat{d} + 2d}{2\tau\theta}. \quad (3.12)$$

We have

$$x_{t+1} = \frac{\partial H_t^S}{\partial P_{t+1}^S}, \quad (3.13)$$

therefore we obtain

$$\begin{aligned} x_{t+1} = & \alpha x_t + (\beta_1 - \beta_2/2 - \beta_3/4) I_t^S + \frac{(-2\beta_2 + \beta_3)(\beta_2 - \beta_3/2)}{2\tau\theta} P_{t+1}^M + \frac{(2\beta_2\beta_3) - (3\beta_3^2)}{4\tau\theta} P_{t+1}^R \\ & + \frac{(2\beta_2 - \beta_3)(1 - \tau + \widehat{d}) + 2\beta_3(1 - \tau)}{4\tau\theta}. \end{aligned} \quad (3.14)$$

We also have

$$\begin{aligned} \mu_{t+1} = & \frac{\partial H_t^S}{\partial P_{t+1}^M} = \alpha \mu_t - \frac{(\beta_2 - \beta_3/2)}{2} I_t^S + \frac{(\beta_2 - \beta_3/2)(\beta_3 - 2\beta_2)}{2\tau\theta} P_{t+1}^S \\ & - \frac{(\beta_2 - \beta_3/2)d}{\tau\theta}. \end{aligned} \quad (3.15)$$

And we obtain

$$P_t^S = \frac{\partial H_t^S}{\partial x_t} = 2\delta x_t + \alpha P_{t+1}^S + 2\widehat{\delta} u_t. \quad (3.16)$$

Since we use open-loop information, the structure variables depend on the time variable and the initial state variables. The x_1 is given initial parameter, $u_1 = 0$ and $\mu_1 = 0$. Furthermore, the boundary condition are $P_{t+1}^R = 0$, $P_{t+1}^M = 0$ and $P_{t+1}^S = 0$.

3.3 Augmented Discrete Hamiltonian Matrix

In this section for solving the optimal control problem formulated in Section 3.1 and 3.2, we chose an algorithm given by Medanic and Radojevic which is based on an augmented discrete Hamiltonian matrix [18]. First, we assume

$$\begin{bmatrix} \widetilde{x}_{t+1} \\ \widetilde{P}_t \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \widetilde{x}_t \\ \widetilde{P}_{t+1} \end{bmatrix} + \begin{bmatrix} D \\ E \end{bmatrix} = \begin{bmatrix} A\widetilde{x}_t + B\widetilde{P}_{t+1} + D \\ C\widetilde{x}_t + A\widetilde{P}_{t+1} + E \end{bmatrix},$$

$$\text{where } \widetilde{x}_{t+1} = \begin{bmatrix} x_{t+1} \\ u_{t+1} \\ \mu_{t+1} \end{bmatrix} \quad \text{and} \quad \widetilde{P}_{t+1} = \begin{bmatrix} p_{t+1}^S \\ p_{t+1}^M \\ p_{t+1}^R \end{bmatrix},$$

A, B , and C are 3×3 matrices, and D and E are 3×1 matrices, such that

$$\begin{aligned} \tilde{x}_{t+1} = \begin{bmatrix} x_{t+1} \\ \mu_{t+1} \\ u_{t+1} \end{bmatrix} &= \begin{bmatrix} A\tilde{x}_t + B\tilde{P}_{t+1} + D \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_t \\ \mu_t \\ u_t \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{13} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} P_{t+1}^S \\ P_{t+1}^M \\ P_{t+1}^R \end{bmatrix} \\ + \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} &= \begin{bmatrix} a_{11}x_t + a_{12}\mu_t + a_{13}u_t + b_{11}P_{t+1}^S + b_{12}P_{t+1}^M + b_{13}P_{t+1}^R + d_1 \\ a_{21}x_t + a_{22}\mu_t + a_{23}u_t + b_{21}P_{t+1}^S + b_{22}P_{t+1}^M + b_{23}P_{t+1}^R + d_2 \\ a_{31}x_t + a_{32}\mu_t + a_{33}u_t + b_{31}P_{t+1}^S + b_{32}P_{t+1}^M + b_{33}P_{t+1}^R + d_3 \end{bmatrix}. \end{aligned} \quad (3.17)$$

The boundary conditions are $\tilde{x}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\tilde{P}_{T+1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

where

$$a_{11} = \alpha, \quad a_{12} = 0, \quad a_{13} = 0, \quad a_{21} = 0, \quad a_{22} = \alpha, \quad a_{23} = 0, \quad a_{31} = 0, \quad a_{32} = 0, \quad a_{33} = \alpha.$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix},$$

where

$$b_{11} = \frac{(\beta_1 - \beta_2/2 - \beta_3/4)(-2\beta_1 + \beta_2 + \beta_3/2)}{\tau\theta}.$$

$$b_{12} = \frac{(\beta_1 - \beta_2/2 - \beta_3/4)(\beta_2 - \beta_3/2)}{\tau\theta} + \frac{(\beta_2 - \beta_3/2)(-2\beta_2 + \beta_3)}{2\tau\theta}.$$

$$b_{13} = \frac{(\beta_1 - \beta_2/2 - \beta_3/4)(\beta_3)}{2\tau\theta} + \frac{(2\beta_2\beta_3)(-3\beta_3^2)}{4\tau\theta}.$$

$$b_{21} = \frac{(\beta_2 - \beta_3/2)(2\beta_1 - 3\beta_2 + \beta_3/2)}{2\tau\theta}.$$

$$b_{22} = -\frac{(\beta_2 - \beta_3/2)^2}{2\tau\theta}.$$

$$b_{23} = -\frac{\beta_3(\beta_2 - \beta_3/2)}{4\tau\theta}.$$

$$b_{31} = -\frac{\beta_3(-2\beta_1 + \beta_2 + \beta_3/2)}{4\tau\theta}.$$

$$b_{32} = \frac{-\beta_3^2 + 3/2\beta_3(\beta_2 - \beta_3/2)}{2\tau\theta}.$$

$$b_{33} = \frac{-\beta_3^2}{8\tau\theta}.$$

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}, \text{ where}$$

$$d_1 = \frac{(\beta_1 - \beta_2/2 - \beta_3/4)(3 - 3\tau - \widehat{d} + 2d)}{2\tau\theta} + \frac{(2\beta_2 - \beta_3)(1 - \tau + \widehat{d}) + 2\beta_3(1 - \tau)}{4\tau\theta}.$$

$$d_2 = \frac{(-\beta_2 - \beta_3/2)(6d - \widehat{d} - 3\tau + 3)}{4\tau\theta}.$$

$$d_3 = \frac{-\beta_3(-7\widehat{d} + 2d - \tau + 1)}{8\tau\theta}.$$

Similarly, we can get the values of the matrices C and E

$$\begin{aligned} \widetilde{P}_t &= \begin{bmatrix} P_t^S \\ P_t^M \\ P_t^R \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} x_t \\ \mu_t \\ u_t \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} P_{t+1}^S \\ P_{t+1}^M \\ P_{t+1}^R \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \\ &= \begin{bmatrix} c_{11}x_t + c_{12}\mu_t + c_{13}u_t + a_{11}P_{t+1}^S + a_{12}P_{t+1}^M + a_{13}P_{t+1}^R + e_1 \\ c_{21}x_t + c_{22}\mu_t + c_{23}u_t + a_{21}P_{t+1}^S + a_{22}P_{t+1}^M + a_{23}P_{t+1}^R + e_2 \\ c_{31}x_t + c_{32}\mu_t + c_{33}u_t + a_{31}P_{t+1}^S + a_{32}P_{t+1}^M + a_{33}P_{t+1}^R + e_3 \end{bmatrix}. \end{aligned} \quad (3.18)$$

$$\text{Therefore } C = \begin{bmatrix} 2\delta & 2\widehat{\delta} & 2\widehat{\widehat{\delta}} \\ 2\widehat{\delta} & 0 & 2\widehat{\widehat{\delta}} \\ 2\widehat{\widehat{\delta}} & 0 & 0 \end{bmatrix} \text{ and } E = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

3.4 Resolution

The above problem is solved by the sweep method [2], by assuming a linear relation between \widetilde{p}_t and \widetilde{x}_t

$$\widetilde{p}_k = S_k \widetilde{x}_k - g_k. \quad (3.19)$$

Thus, we can compute

$$\widetilde{\widetilde{x}_{k+1}} = (I_{2*2} - BS_{k+1})^{-1}(A\widetilde{x}_t - Bg_{k+1} + D). \quad (3.20)$$

Then by substituting (3.19) and (3.20) into the definition of p_{k+1} as given by the augmented Hamiltonian matrix, and equating both sides we finally get the difference equations:

$$S_k = C + AS_{k+1}(I_{2*2} - BS_{k+1})^{-1}. \quad (3.21)$$

$$g_k = AS_{k+1}(I_{2*2} - BS_{k+1})^{-1} + Bg_{k+1} - D + Ag_{k+1} - E. \quad (3.22)$$

The boundary conditions are $\tilde{x}_1 = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix}$ and $\tilde{P}_{T+1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. And then $S_{T+1} = 0_{3*3}$ and $g_{T+1} = 0_{3*1}$.

From the boundary conditions we get $S_T = C$ and $g_T = E$. Once we get the different values of S_k and g_k by the backward loop, then the values of \tilde{x}_t and \tilde{p}_t are computed by a forward loop. And, consequently we get the values of $x_t, I_t^S, I_t^M, I_t^R, p_t^S, p_t^M, p_t^R$, for all points in time.

4 Numerical Example

In this section we provide a numerical example. We run the following numerical simulations with mathematica 8. The results presented here are obtained for the following values of the parameters: $a = 6, w = 3.8, c = 2.4, q_t = 100000, d = 0.6, c = 0.00001, d = 0.4, \hat{d} = 0.4, z = 6, \theta = 0.01$, and $\tau = 0.2$. We set $\beta_1 = 0.3, \beta_2 = 0.5$ and $\beta_3 = 0.8$; and $\beta_1 = 0.3, \beta_2 = 0.5$. $B_t(x_t) = \delta x_t$, the potential benefits firms obtain from taking social responsibility, such as increased demand, better reputation and so on. We set $\delta = 0.2, \hat{\delta} = 0.2$ and $\hat{\hat{\delta}} = 0.2$. We assume that the time horizon is $T=10$. The initial level of social responsibility is supposed to be $x_1=1$. We draw the results of the equilibrium from our model, a three-stage Stackelberg dynamic game.

The figure 3 shows the trend of profits from periods one to ten in a Stackelberg game. JS is the supplier's profit, JM is manufacturer's profit and JR is retailer's profit. We compare the profits of the supplier, manufacturer and retailer over a time horizon, first while playing the game and then, without playing the game. Figure 4 shows the difference in supplier's profits when playing the game and without playing. JSO is supplier's profit without playing the game; JS is supplier's profit when playing the game. As in the first graph, the second and third one (figure 5, 6) shows the difference in manufacturer's profit and retailer's profits when playing the game and without playing. JMO is manufacturer's profit without playing the game; JM is manufacturer's profit when playing the game. JRO is retailer's profit without playing the game; JR is retailer's profit when playing the game. Obviously, all of players gain extra profit from playing the games. Figure 7 compares the cumulated profits of the member's of supply chain, playing game one and without playing game.

In sum, the supplier, manufacturer and retailer are motivated to play the game because their respective benefit increases and the supplier as the leader in the game earns more benefit than the

followers. Of course, this result is obtained with a very specific dynamic game model. Another one may give different results.

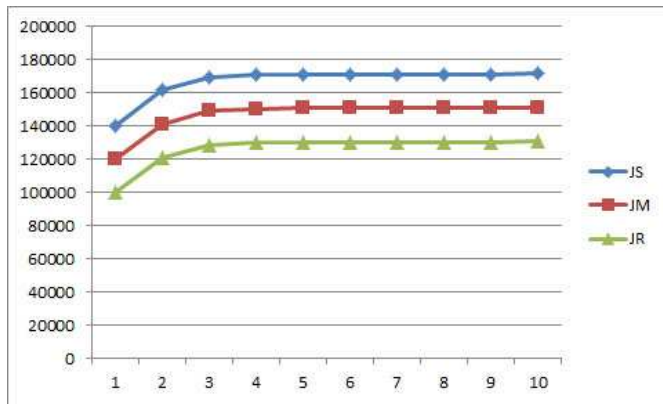


Figure 3: Profits of supplier, manufacturer and retailer.

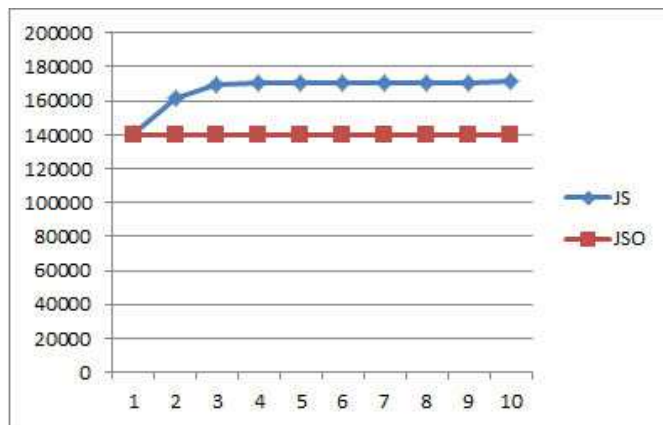


Figure 4: Comparison of the supplier's profit, playing game one and without playing any game.

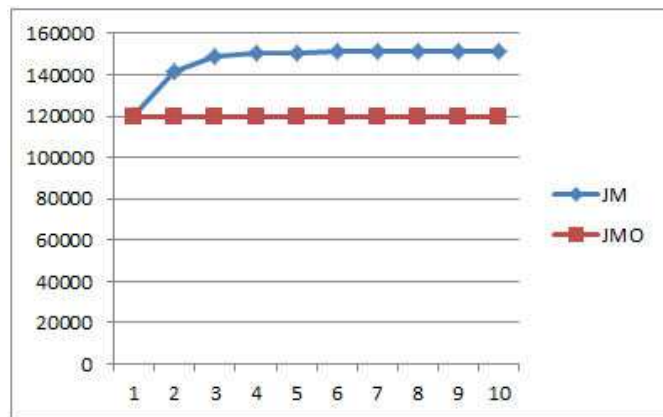


Figure 5: Comparison of the manufacturer's profit, playing game one and without playing any game.

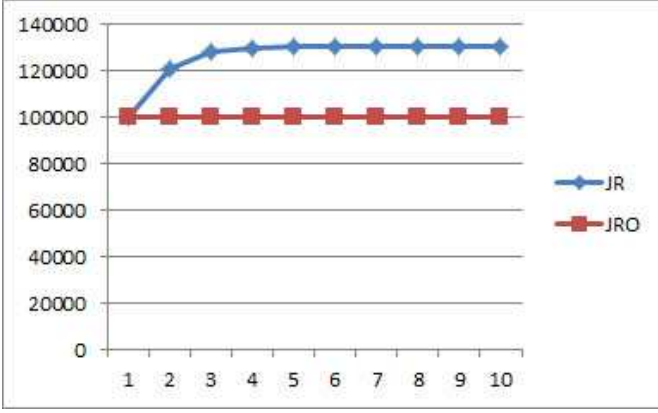


Figure 6: Comparison of the retailer's profit, playing game one and without playing any game.

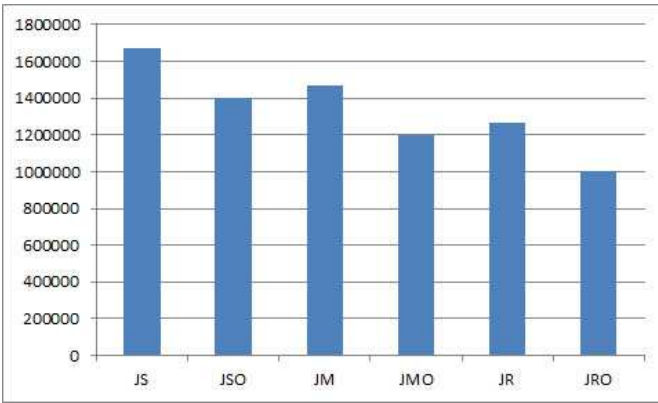


Figure 7: Comparison of the cumulated profits of the member's of supply chain, playing game one and without playing any game.

5 Conclusion

In this paper we investigated a decentralized three-tire supply chain consisting of supplier, manufacturer and retailer with the aim of allocating CSR to members of the supply chain system over time. We considered two-level Stackelberg game consisting of two followers and one leader. The members of a supply chain play games with each other to maximize their own profits; thus, the model used was a long-term co-investment game model. The equilibrium point in a time horizon was determined at where the profit of supply chain's members was maximized and CSR was implemented among members of the supply chain. We applied control theory and used an algorithm (augmented discrete Hamiltonian matrix) to obtain an optimal solution for the dynamic game model. We presented a numerical example and we found that, the benefits of the player increased when they played the game.

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